

CHS Algebra 2 Summer Packet

Hello, incoming Algebra 2 student! This summer packet contains topics that you have likely seen in prior math classes. While your Algebra 2 teacher may spend time reviewing some of these topics, your Algebra 2 teacher will also assume that you have an understanding of these topics, and that you remember how to handle the kinds of problems below. If there is a topic that does not seem familiar to you, please look through the worked out examples that are provided for each topic before you try the related problems.

The answers to all problems are provided separately on the school website. Close to the start of the year, your teacher will assess you on these topics. Your teacher will give you more information on how that will work when you meet them in September. Your teacher will spend some class time reviewing these topics at the start of the year, but you may also need to see your teacher outside of class if you have a lot of questions.

You are encouraged to begin looking through this material over the summer, and to see your teacher for help on these topics as needed. Taking these actions are a great way to make sure that you start off strong in Algebra 2!

Solving Multi-Step Equations

1. Clear parentheses using the distributive property.
2. Combine like terms within each side of the equal sign.
3. Add/subtract terms to both sides of the equation to get the terms with variables on one side and constant terms on the other side.
4. Isolate the variable by multiplying/dividing both sides of the equation by the number with the variable.

Ex: $3(2x - 5) - 3 = 2x + 8 + 6x$

$$6x - 15 - 3 = 2x + 8 + 6x$$

$$6x - 18 = 8x + 8$$

$$\begin{array}{r} 6x - 18 = 8x + 8 \\ \underline{-8} \qquad \underline{-8} \\ 6x - 26 = 8x \end{array}$$

$$\begin{array}{r} 6x - 26 = 8x \\ \underline{-6x} \qquad \underline{-6x} \end{array}$$

$$\begin{array}{r} -26 = 2x \\ \underline{2} \qquad \underline{2} \end{array}$$

$$-13 = x \rightarrow \boxed{x = -13}$$

Solving Absolute Value Equations

1. Isolate the absolute value.
2. Break the absolute value equation into two separate equations. For the first equation, set the expression inside the absolute value notation equal to the opposite side of the equation. For the second equation, make the number on the opposite side negative.
3. Solve each equation.

Ex: $-3|3x+2| - 2 = -8$

$$\begin{array}{r} -3|3x+2| - 2 = -8 \\ \underline{+2} \qquad \underline{+2} \end{array}$$

$$\begin{array}{r} -3|3x+2| = -6 \\ \underline{-3} \qquad \underline{-3} \end{array}$$

$$|3x+2| = 2$$

$$3x + 2 = 2$$

$$\downarrow$$
$$x = 0$$

$$3x + 2 = -2$$

$$\downarrow$$
$$x = -\frac{4}{3}$$

$$\boxed{x = \{0, -\frac{4}{3}\}}$$

Solving Word Problems Algebraically

1. Define a variable.
2. Write an equation.
3. Solve the equation.
4. Label your answer with the appropriate units.

Ex: Bobby is 4 years younger than twice Jimmy's age. If Bobby is 26 years old, how old is Jimmy?

Let j = Jimmy's age

$$2j - 4 = 26$$

$$j = 15$$

→ Jimmy is 15 years old

Solve each equation.

| | | |
|--|---------------------------------|------------------------------------|
| 1. $-3x - 9 = -27$ | 2. $25 + 2(n + 2) = 30$ | 3. $-9b - 6 = -3b + 48$ |
| 4. $5 - (m - 4) = 2m + 3(m - 1)$ | 5. $-24 - 10k = -8(k + 4) - 2k$ | 6. $f - (-19) = 11f + 23 - 20f$ |
| 7. $\frac{3}{4}d - \frac{1}{2} = \frac{3}{8} + \frac{1}{2}d$ | 8. $-0.5g + 13 = 3g$ | 9. $-5(h + 12) - (4h - 2) = h - 8$ |
| 10. $ 3x + 4 = 16$ | 11. $3 x - 5 = 27$ | 12. $-8 2x - 6 + 4 = -60$ |

Solve each word problem algebraically.

| | |
|--|--|
| 13. The sum of two consecutive integers is one less than three times the smaller integer. Find the two integers. | 14. The length of a rectangular picture is 5 inches more than three times the width. Find the dimensions of the picture if its perimeter is 74 inches. |
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Solving & Graphing Inequalities

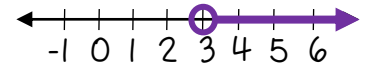
1. Solve the inequality as if it is an equation.
2. If you multiply or divide both sides of the inequality by a negative number, flip the inequality sign.
3. Write your answer with the variable on the left of the inequality sign.
4. Graph the solution on a number line. Make an open circle on the number if the number is not included in the solution ($<$ or $>$) and make a closed circle if the number is included (\leq or \geq). Shade to the left for less than ($<$ or \leq) and shade to the right for greater than ($>$ or \geq).

Ex: $-24 > 3x - 6 - 9x$

$$\begin{array}{r} -24 > -6x - 6 \\ +6 \quad \quad +6 \end{array}$$

$$\begin{array}{r} -18 > -6x \\ -6 \quad -6 \end{array}$$

$$3 < x \rightarrow \boxed{x > 3}$$



Compound Inequalities

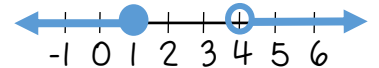
"Or" Inequalities

1. Solve each inequality separately and graph the solution to each on one number line.

Ex: $x + 2 > 6$ or $-2x \geq -2$

$$\begin{array}{r} x + 2 > 6 \\ -2 \quad -2 \end{array} \quad \text{or} \quad \begin{array}{r} -2x \geq -2 \\ -2 \quad -2 \end{array}$$

$$\boxed{x > 4} \quad \text{or} \quad \boxed{x \leq 1}$$



"And" Inequalities:

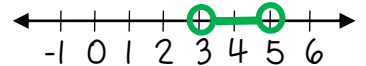
1. Isolate the variable, making sure to do the same thing to all 3 parts of the inequality.
2. Graph the solution to each part of the compound inequality and see where those graphs overlap. The overlapping part is the solution.

Ex: $3 < 2x - 3 < 7$

$$\begin{array}{r} 3 < 2x - 3 < 7 \\ +3 \quad \quad +3 \quad +3 \end{array}$$

$$\begin{array}{r} 6 < 2x < 10 \\ 2 \quad 2 \quad 2 \end{array}$$

$$\boxed{3 < x < 5}$$



Solve each inequality. Graph the solution on a number line.

15. $-6x + 3 > -39$

16. $25 - 3(n - 2) \geq -8n + 6$

17. $8g - 6(g + 1) < 4(2g - 9)$

18. $7k + 1 \leq 8$ or $-7 < k - 10$

19. $-4 < 3b + 2 \leq 20$

20. $9 < -3m < 24$

21. $y + (-6) \geq -13$ or $-3y + 8 > -7$

Finding Slope from 2 Points

Slope Formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Ex: Find the slope of the line that passes through the points $(-9, -3)$ and $(7, -7)$

Special Cases:

$\frac{0}{\#} \rightarrow$ slope = 0

$\frac{\#}{0} \rightarrow$ slope is undefined

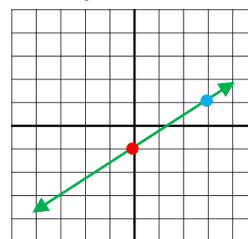
$$m = \frac{-7 - (-3)}{7 - (-9)} = \frac{-4}{16} = \boxed{-\frac{1}{4}}$$

Slope-Intercept Form

$$y = mx + b$$

$m =$ slope & $b =$ y-intercept

Ex: Graph $y = \frac{2}{3}x - 1$



y-intercept is -1
slope = $\frac{2}{3}$, (so from the y-intercept go up 2 & right 3)

Graphing from Slope-Intercept Form:

1. Make a point at the y-intercept.
2. Use the slope ($\frac{\text{rise}}{\text{run}}$) to make more points.
3. Connect the points to form a line.

Standard Form

$$Ax + By = C$$

$A, B,$ & C are integers & A is not negative

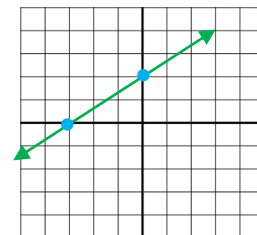
Ex: Graph $2x - 3y = -6$

Graphing Using Intercepts:

1. Find the x-intercept by substituting 0 for y .
2. Find the y-intercept by substituting 0 for x .
3. Make a point at each intercept and then connect the points to form a line.

x-intercept: $2x - 3(0) = -6$
 $2x = -6 \rightarrow x = -3$
 $(-3, 0)$

y-intercept: $2(0) - 3y = -6$
 $-3y = -6 \rightarrow y = 2$
 $(0, 2)$



Point-Slope Form

$$y - y_1 = m(x - x_1)$$

$m =$ slope & (x_1, y_1) is a point on the graph

Ex: Write the equation of the line passing through the points $(-1, 2)$ and $(3, 4)$ in point-slope form. Then convert it to slope-intercept and standard form.

$$m = \frac{4 - 2}{3 - (-1)} = \frac{2}{4} = \frac{1}{2}$$

Point-Slope Form: $y - 2 = \frac{1}{2}(x + 1)$

Converting Point-Slope Form to Slope-Intercept Form:

1. Distribute m .
2. Move y_1 to the other side of the equation.

Convert to Slope-Intercept Form:

$$\rightarrow y - 2 = \frac{1}{2}x + \frac{1}{2} \rightarrow y = \frac{1}{2}x + \frac{5}{2}$$

Converting Slope-Intercept Form to Standard Form:

1. Bring the x term to the left.
2. If there are fractions in the equation, multiply everything through by the least common denominator.
3. If A is negative, multiply everything through by -1 .

Convert to Standard Form:

$$\rightarrow -2\left(-\frac{1}{2}x + y = \frac{5}{2}\right) \rightarrow \boxed{x - 2y = -5}$$

Find the slope of the line that passes through the pair of points.

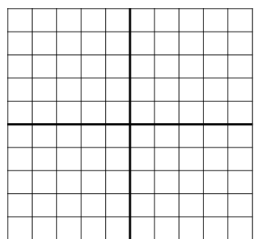
25. $(9, -3)$ and $(9, -8)$

26. $(-8, 5)$ and $(3, -6)$

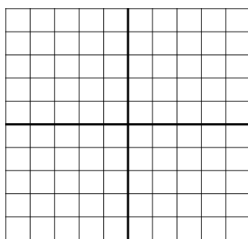
27. $(7, -1)$ and $(15, 9)$

Graph each line.

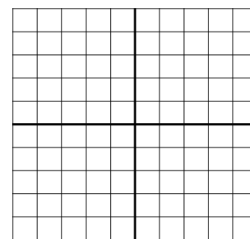
28. $y = -\frac{3}{2}x + 2$



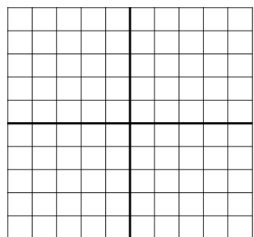
29. $y = x - 3$



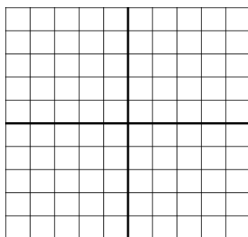
30. $y = \frac{1}{3}x + 5$



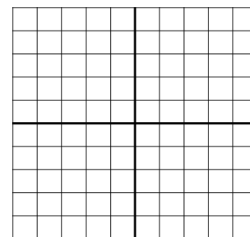
31. $2x - y = -2$



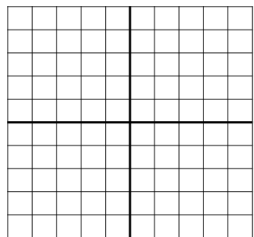
32. $x + y = 4$



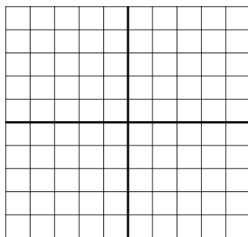
33. $3x + 4y = -12$



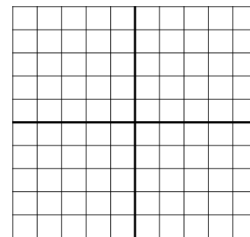
34. $y + 3 = \frac{1}{2}(x + 2)$



35. $y - 1 = \frac{2}{3}(x - 3)$



36. $y - 2 = 0$



Write the equation of the line in point-slope, slope-intercept, and standard form.

37. Line passing through point $(3, 5)$ with a slope of 1

38. Line passing through points $(-4, 2)$ and $(0, 3)$

39. Line passing through points $(1, 3)$ and $(2, 5)$

Parallel & Perpendicular Lines

Parallel Lines have the *same slope* but different y-intercepts.

Perpendicular Lines have *opposite reciprocal slopes*.

Writing Equations of Parallel Lines:

1. Find the slope of the original line by first converting it to slope-intercept form if it is in Standard Form. The slope of the line parallel will have that same slope.
2. Use the given point along with the slope you just found to write the equation of the line in point-slope form.
3. Convert the point-slope form equation to slope-intercept form.

Ex: Write the equation of the line that is parallel to the line $y = 3x - 5$ and passes through the point $(-2, 4)$.

$$y = 3x - 5$$

$m = 3$, so slope of parallel line is 3 , too

$$\rightarrow y - 4 = 3(x + 2)$$

$$\rightarrow y - 4 = 3x + 6$$

$$\rightarrow \boxed{y = 3x + 10}$$

Ex: Write the equation of the line that is perpendicular to the line $x - 3y = -6$ and passes through the point $(-1, 1)$.

$$x - 3y = -6 \rightarrow -3y = -x - 6$$

$$\rightarrow y = \frac{1}{3}x + 2$$

$m = \frac{1}{3}$, so slope of perpendicular line is -3

$$\rightarrow y - 1 = -3(x + 1)$$

$$\rightarrow y - 1 = -3x - 3$$

$$\rightarrow \boxed{y = -3x - 2}$$

Writing Equations of Perpendicular Lines:

1. Find the slope of the original line. The slope of the line perpendicular will have the opposite (negative) reciprocal slope.
2. Use the given point along with the slope you just found to write the equation of the line in point-slope form.
3. Convert the point-slope form equation to slope-intercept form.

Linear Inequalities

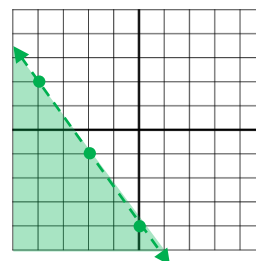
1. Convert the linear inequality in slope-intercept form. Be sure the y is on the left and remember to flip the inequality sign if you multiply or divide by a negative.
2. Graph the line as if it is an equation, except use a dotted line if the inequality sign is $<$ or $>$. If the sign is \leq or \geq , use a regular solid line.
3. Shade above the line for a "greater than" inequality ($>$ or \geq). Shade below the line for a "less than" inequality ($<$ or \leq). (For vertical lines, shade to the right for greater than and to the left for less than).

Ex: $-3x - 2y > 8$

$$\begin{array}{r} -3x - 2y > 8 \\ +3x \qquad +3x \end{array}$$

$$\frac{-2y}{-2} > \frac{3x + 8}{-2}$$

$$y < -\frac{3}{2}x - 4$$



Determine whether the lines are parallel, perpendicular, or neither. Justify your answer.

$$40. \begin{aligned} y &= 2x - 8 \\ y &= \frac{1}{2}x + 6 \end{aligned}$$

$$41. \begin{aligned} y &= x \\ x + y &= -2 \end{aligned}$$

$$42. \begin{aligned} 3x + 2y &= 18 \\ y + 4 &= -\frac{3}{2}(x - 4) \end{aligned}$$

Write the equation of the line parallel to the given line that passes through the given point in slope-intercept form.

$$43. y = -4x - 2; (0, -1)$$

$$44. 2x - y = -4; (2, 5)$$

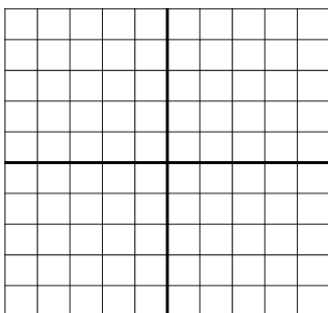
Write the equation of the line perpendicular to the given line that passes through the given point in slope-intercept form.

$$45. y = \frac{2}{3}x - 9; (-6, -2)$$

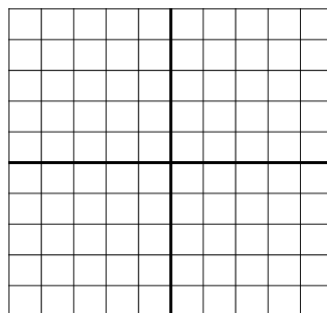
$$46. 4x + y = -6; (4, 5)$$

Graph the solution to each linear inequality.

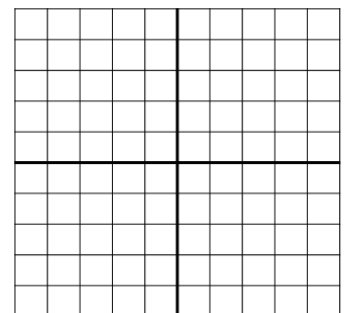
$$47. y \leq -4x - 3$$



$$48. 2x - y < 1$$



$$49. x + 3y > 3$$



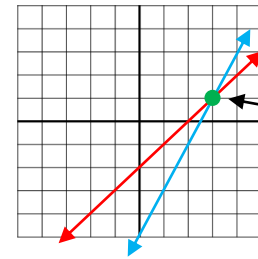
Solving Systems of Equations by Graphing

1. Graph both lines on the same coordinate plane.
2. Find the point where the lines meet, and write that solution as an ordered pair.

Special Cases:

- parallel lines: no solution
- coincident lines (lines that are the same): infinitely many solutions

Ex: Solve the system by graphing: $\begin{cases} y = x - 2 \\ y = 2x - 5 \end{cases}$



solution:
 $(3, 1)$

Solving Systems of Equations Using Substitution

1. Solve one of the equations for x or y.
2. Replace the x or y in the other equation with the expression you found in step 1 that equals that variable.
3. Solve the equation.
4. Substitute the solution you found in step 3 with the variable in your step 1 equation to solve for the other variable.
5. Write your solution as an ordered pair.

Ex: Solve the system by substitution: $\begin{cases} x + 3y = 4 \\ 2x - 3y = -1 \end{cases}$

$$\begin{aligned} x + 3y = 4 &\rightarrow x = -3y + 4 \\ 2x - 3y = -1 &\rightarrow 2(-3y + 4) - 3y = -1 \\ &\rightarrow -6y + 8 - 3y = -1 \\ &\rightarrow -9y + 8 = -1 \\ &\rightarrow -9y = -9 \rightarrow y = 1 \\ \rightarrow x = -3y + 4 &\rightarrow x = -3(1) + 4 \rightarrow x = 1 \\ \text{solution: } &(1, 1) \end{aligned}$$

Solving Systems of Equations Using Elimination

1. Write both equations in Standard Form.
2. Multiply neither, one, or both of the equations by constants so that either the x coefficients or the y coefficients are opposites (i.e. 2 and -2).
3. Add the two equations. The terms with the opposite coefficients will cancel out.
4. Solve the equation for the variable that didn't cancel out.
5. Substitute the solution you found in step 4 for the variable in any of the equations, and solve to find the other variable.
6. Write your solution as an ordered pair.

Ex: Solve the system by elimination: $\begin{cases} 3x + 4y = 2 \\ -2x + 2y = -6 \end{cases}$

Multiplying by -2 will give the y terms opposite coefficients

$$\begin{aligned} &3x + 4y = 2 \\ &-2(-2x + 2y = -6) \\ \rightarrow &+ \begin{array}{r} 3x + 4y = 2 \\ 4x - 4y = 12 \\ \hline 7x = 14 \end{array} \rightarrow x = 2 \end{aligned}$$

$$\begin{aligned} \rightarrow 3x + 4y = 2 &\rightarrow 3(2) + 4y = 2 \\ \rightarrow 6 + 4y = 2 &\rightarrow 4y = -4 \rightarrow y = -1 \\ \text{solution: } &(2, -1) \end{aligned}$$

Systems of Equations Word Problems

1. Define 2 variables.
2. Write 2 equations.
3. Solve the system of equations using the method of your choice.
4. Label your solution with the appropriate units.

Ex: A 24 question test contains some 3 point questions and some 5 point questions. If the test is worth 100 points, how many of each type of questions are there?

Let $x = \#$ of 3 point questions
 $y = \#$ of 5 point questions

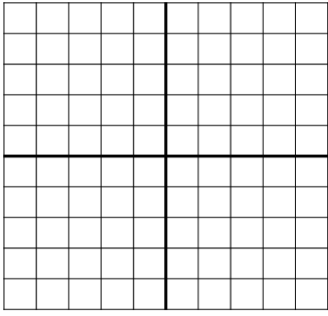
$$\begin{aligned} x + y &= 24 \\ 3x + 5y &= 100 \end{aligned}$$

solve using substitution or elimination \rightarrow solution: (10, 14)

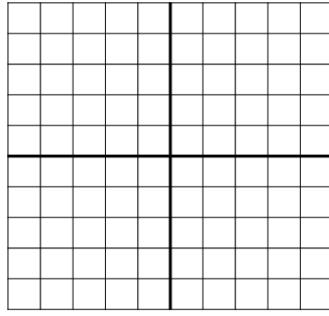
\rightarrow There were 10 3-point questions and 14 5-point questions.

Solve each system of equations by graphing.

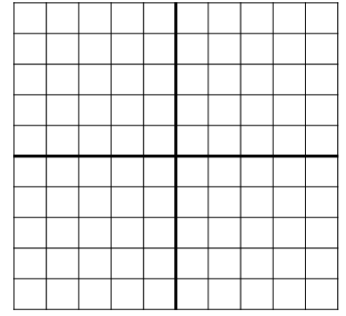
$$50. \begin{cases} y = \frac{1}{2}x - 4 \\ y = -x - 1 \end{cases}$$



$$51. \begin{cases} y = 2x + 1 \\ -y = -2x + 1 \end{cases}$$



$$52. \begin{cases} x - 2y = 4 \\ -3x + 2y = -8 \end{cases}$$



Solve each system of equations using substitution.

$$53. \begin{cases} y = 2x + 3 \\ 5x - 2y = -6 \end{cases}$$

$$54. \begin{cases} x + 4y = 5 \\ -2x + 5y = 16 \end{cases}$$

$$55. \begin{cases} 9y - 7x = -13 \\ -9x + y = 15 \end{cases}$$

Solve each system of equations using elimination.

$$56. \begin{cases} 3x - 7y = -29 \\ -4x + 7y = 27 \end{cases}$$

$$57. \begin{cases} -4x - 8y = -48 \\ 8x + 3y = -34 \end{cases}$$

$$58. \begin{cases} 3x - 7y = 21 \\ 6x = 14y + 42 \end{cases}$$

Solve each word problem using a system of equations.

59. Joe bought 5 apples and 4 bananas for \$6. Dawn bought 3 apples and 6 bananas for \$6.30. How much does each apple and each banana cost?

60. Wesley and Brian have a total of 87 baseball cards. Wesley has 30 less than twice as many cards as Brian. How many baseball cards do they each own?

Exponent Rules

Zero Exponent: Any base raised to the zero power equals 1.

$$\text{Ex: } (-9)^0 = 1$$

Negative Exponent: Move the base to the opposite side of the fraction bar and make the exponent positive.

$$\text{Ex: } 3^{-4} = \frac{1}{3^4} = \frac{1}{81}$$

Monomial x Monomial: Multiply the coefficients and add the exponents of like bases.

$$\text{Ex: } (-2x^3)(8x^{-5}) = -16x^{-2} = \frac{-16}{x^2}$$

Monomial ÷ Monomial: Divide the coefficients and subtract the exponents of like bases.

$$\text{Ex: } \frac{4ab^3}{4a^2b^2} = |a^{-1}b| = \frac{b}{a}$$

Power of a Monomial: Raise each base (including the coefficient) to that power. If a base already has an exponent, multiply the two exponents.

$$\text{Ex: } (3x^3y^2)^3 = 3^3x^9y^6 = 27x^9y^6$$

Power of a Quotient: Raise each base (including the coefficients) to that power. If a base already has an exponent, multiply the two exponents.

$$\text{Ex: } \left(\frac{5a^3b}{2c^{-1}}\right)^2 = \frac{5^2a^6b^2}{2^2c^{-2}} = \frac{25a^6b^2c^2}{4}$$

Simplify each expression completely. Write your answer using only positive exponents.

61. $x^6 \cdot x^4$

62. $(5^3)^2$

63. $-6a^2b^{-4}c \cdot 4ab^2$

64. $\frac{a^3b^{-6}}{c^{-2}}$

65. $\left(\frac{-2x^6y}{3z^5}\right)^3$

66. $(8w^3q^{-5})^0$

67. $\frac{24d^5f^{-5}g^8}{36d^{-3}f^9g^2}$

68. $(2b^{-3}d^6)^4 \cdot 3b^7d$

69. $\left(\frac{-4a^4b^2c^{-1}}{6a^9}\right)^{-1}$

Classifying Polynomials

- Term: each part of a polynomial separated by addition or subtraction
- Degree of a Term: the sum of the exponents of the variables in a term
- Degree of Polynomial: the highest degree of all the terms in a polynomial

Ex: Classify $3x^3 - 9x + 7$.

It is a trinomial because there are 3 terms separated by - and +

The degree of the 1st term is 3, the degree of the 2nd term is 1, and the degree of the 3rd term is 0. So, the degree of the polynomial is 3 since that is the highest degree of all the terms.

Classifying By Number of Terms:

- 1 term: monomial
- 2 terms: binomial
- 3 terms: trinomial
- ≥ 4 terms: n-term polynomial

Classifying Polynomials By Degree:

- 0: constant
- 1: linear
- 2: quadratic
- 3: cubic
- 4: quartic
- 5: quintic
- ≥ 6: nth degree

→ Is it a cubic trinomial

Adding & Subtracting Polynomials

Adding Polynomials:

1. Add like terms together.
2. Write your answer in Standard Form (decreasing order of degree).

Ex: $(4x^2 - 9) + (7x - 9x^2 + 8)$

$$(4x^2 - 9) + (7x - 9x^2 + 8)$$

$$= -5x^2 - 1 + 7x \rightarrow -5x^2 + 7x - 1$$

Subtracting Polynomials:

1. Turn into an addition problem by changing the - to + between the two polynomials and reversing the sign of each term in the second polynomial.
2. Add like terms together.
3. Write your answer in Standard Form.

Ex: $(3x^2 - 6x - 9) - (2x^2 + 8x - 3)$

$$\rightarrow (3x^2 - 6x - 9) + (-2x^2 - 8x + 3)$$

$$= x^2 - 14x - 6$$

Multiplying Polynomials

Monomial x Polynomial:

1. Use the Distributive Property to multiply the monomial by each term.
2. Write your answer in Standard Form.

Ex: $4x^2(3x^2 - 8x - 5)$

$$4x^2(3x^2 - 8x - 5)$$

$$= 12x^4 - 32x^3 - 20x^2$$

Binomial x Binomial:

1. FOIL (multiply the two first terms, the two outer terms, the two inner terms, and the two last terms).
2. Combine like terms and write your answer in Standard Form.

Ex: $(x + 3)(2x - 1)$

$$(x + 3)(2x - 1)$$

F: $2x^2$ O: $-1x$ I: $6x$ L: -3

$$= 2x^2 + 5x - 3$$

Any Polynomial x Any Polynomial:

1. Multiply each term from the first polynomial by each term in the second polynomial.
2. Combine like terms and write your answer in Standard Form.

Ex: $(x + 2)(x^2 - 3x - 8)$

$$(x + 2)(x^2 - 3x - 8)$$

$$= x^3 - 3x^2 - 8x + 2x^2 - 6x - 16$$

$$= x^3 - x^2 - 14x - 16$$

Classify each polynomial by its degree and number of terms.

| | | | |
|-----------------|----------------------|---------------|--------------|
| 76. $8x^3 - 9x$ | 77. $-2 - 4x^2 + 7x$ | 78. $8x^2y^2$ | 79. $6x + 5$ |
|-----------------|----------------------|---------------|--------------|

Find each sum or difference. Write your answer in Standard Form.

| | | |
|-------------------------------------|------------------------------------|--------------------------------------|
| 80. $(2h^3 + 6h) + (3h^3 - 7h - 3)$ | 81. $(8x - 4x^2 + 3) - (7x^2 - 9)$ | 82. $(-14a^2 - 5) - (5a^2 + 6a - 7)$ |
|-------------------------------------|------------------------------------|--------------------------------------|

Find each product. Write your answer in Standard Form.

| | | |
|-----------------------------|----------------------|--------------------------|
| 83. $5x^3(9x^2 + 4x - 5)$ | 84. $(x + 4)(x - 3)$ | 85. $(3n - 8)(4n - 7)$ |
| 86. $(2x + 3)(x^2 + x + 3)$ | 87. $(6x + 1)^2$ | 88. $4g(2g - 9)(2g + 9)$ |

Simplify each expression completely. Write your answer in Standard Form.

| | |
|--|----------------------------------|
| 89. $(x + 2)(x + 8) + (4x^2 + 8x - 3)$ | 90. $(x + 5)(x - 5) - 6x(x + 1)$ |
|--|----------------------------------|

Factoring Out a GCF

1. Find the largest monomial that is a factor of each term in the polynomial, and pull it out in front of parentheses.
2. Divide each term by the GCF and write the resulting polynomial in the parentheses.

Ex: Factor $25x^4y - 30x^3y^2 + 10x^2y^3$

GCF = $5x^2y$, so divide each term by $5x^2y$

$$\rightarrow 5x^2y(5x^2 - 6xy + 2y^2)$$

Factoring 4-Term Polynomials

First factor out a GCF if there is one. Then factor by grouping as described below.

Factor by Grouping

1. Group the first two terms in parentheses and the last two terms in parentheses.
2. Factor out the GCF from each set of parentheses. (The two resulting binomials in parentheses should match).
3. Factor out the common binomial.

Ex: Factor $3x^3 - 6x^2 + 5x - 10$

$$\begin{aligned} &(3x^3 - 6x^2) + (5x - 10) \\ \rightarrow &3x^2(x - 2) + 5(x - 2) \\ \rightarrow &(x - 2)(3x^2 + 5) \end{aligned}$$

Factoring Binomials

First factor out a GCF if there is one. Then determine whether it is a difference of squares binomial (in the form $a^2 - b^2$). If it is, use the method below.

Binomials in the form $a^2 - b^2$

1. Find the square root of the first term (a) and the square root of the second term (b). Your answer will be $(a + b)(a - b)$.

Ex: Factor $16x^2 - 25$

The square root of $16x^2 = 4x$ & the square root of $25 = 5$

$$\rightarrow (4x + 5)(4x - 5)$$

Factoring Trinomials

First factor out a GCF if there is one. Then use the appropriate method below, depending on whether or not the leading coefficient is 1.

Trinomials in the form $x^2 + bx + c$ (leading coefficient = 1)

1. Find two numbers with a product of c and a sum of b .
2. Your answer will be written as the product of two binomials: $(x + 1^{\text{st}} \text{ number})(x + 2^{\text{nd}} \text{ number})$.

Ex: Factor $x^2 - 6x + 8$

Need 2 numbers with product of 8 and sum of -6.
 \rightarrow the 2 numbers are -4 & -2

$$\rightarrow (x + -4)(x + -2) \rightarrow (x - 4)(x - 2)$$

Trinomials in the form $ax^2 + bx + c$ (leading coefficient > 1)

1. Multiply a and c . Find two numbers with a product of ac and a sum of b .
2. Copy the ax^2 term from the original trinomial, and then split up the bx term into two terms, using the two numbers you found in step 1 as the coefficients of each term. Copy the c term from the original trinomial. (So now you have a 4-term polynomial).
3. Factor by grouping.

Ex: Factor $2x^2 + 7x + 3$

$ac = 2 \cdot 3 = 6$. Need 2 numbers with a product of 6 and a sum of 7.

\rightarrow the 2 numbers are 6 & 1

$$\begin{aligned} \rightarrow &2x^2 + 6x + 1x + 3 \\ \rightarrow &(2x^2 + 6x) + (1x + 3) \\ \rightarrow &2x(x + 3) + 1(x + 3) \\ \rightarrow &(x + 3)(2x + 1) \end{aligned}$$

Factor each polynomial completely.

91. $-18x - 27$

92. $x^2 - 100$

93. $x^2 - 5x + 6$

94. $2x^2 + 7x + 6$

95. $5x^3 + 3x^2 + 10x + 6$

96. $3x^2 - 12$

97. $x^2 + 24x + 144$

98. $9x^3 - 30x^2 - 24x$

99. $8x^3 + 4x^2 - 6x - 3$

100. $5x^2 + 10x - 45$

101. $36x^4 - 121$

102. $5x^2 + 22x + 8$

103. $4x + 16xy + 9y + 36y^2$

104. $x^2 - 3x - 88$

105. $4x^2 - 15x + 9$