Pre-Calc Unit Lesson #2: Interval Notation/Inequalities

Change the following to interval notation:

1. $-2 \leq x < 3$ 2. $x \leq 3$ or $x > 5$ 3. All real, $x \neq 2$ 4. $x < 3$ and $x > -5$

$[-2,3)$ $(-\infty,3] \cup (5,\infty)$ $(-\infty,2) \cup (2,\infty)$ $(-5,3)$

Make sure you draw a picture for #2 and #4 just in case it can simplify. In #4, two "and graphs" overlapped and we were describing a set of numbers that was both less than 3 and greater than –5 at the same time. If there is an "and", it should always be simplified before changing to the other notation.

Change the following to inequality notation: (You may need to draw a number line to help you.)

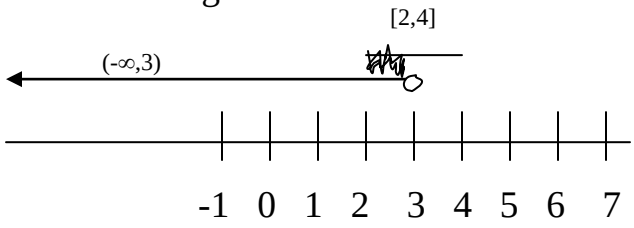
1. $(-\infty,2) \cup [5,\infty)$

$x < 2$ or $x \geq 5$

2. $(-\infty,3) \cap [2,4]$

$2 \leq x < 3$

Notice the overlap is from x = 2 to x = 3, not including x = 3



Solve the following inequalities:

1. $x - 4x - 5 > 0$ This is a linear inequality and can be solved like a regular linear equation with one exception: If you divide or multiply by a negative, the inequality sign must be flipped.

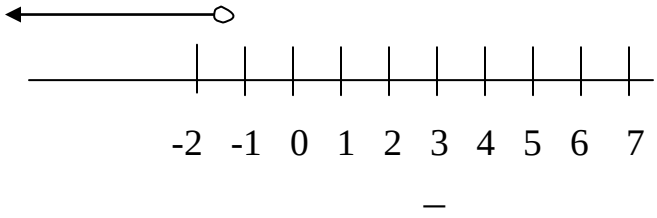
$x - 4x - 5 > 0$

$-3x - 5 > 0$

$-3x > 5$

$x < -5/3$ Note: sign is flipped since we divided by –3 on both sides

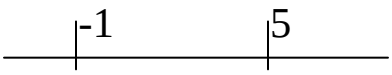
Answers:

Inequality	Interval	Graph
$x < -5/3$	$(-\infty,-5/3)$	

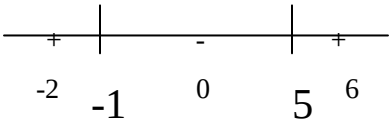
Note the open circle on the graph and that –5/3 is approximately –1.6

2. $x^2 - 4x - 5 > 0$ This is a quadratic inequality and requires a number line, using the roots of the equation $x^2 - 4x - 5 = 0$ and then picking values in between to determine the region that works.

$(x - 5)(x + 1) > 0$ So, we use x = 5 and x = -1:



Plug in numbers that are on the three regions into the factored function and see if the value is positive or negative.



(-2 – 5)(-2 + 1) = (-)(-) = +
(0 – 5)(0 + 1) = (-)(+) = -
(6 – 5)(6 + 1) = (+)(+) = +

The value picked from the first region is positive and thus the whole region is positive. Our original problem was asking when this inequality was greater than zero and so we choose the first and third regions. Also note that the answer should include open circles.

Inequality: x < -1 or x > 5 Interval: $(-\infty,-1) \cup (5,\infty)$

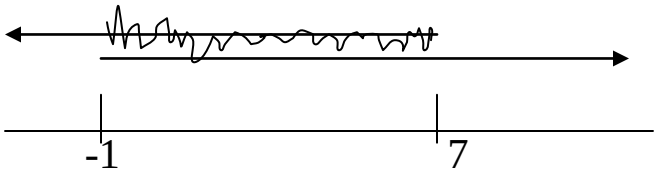
3. $|x-3| \leq 4$ This is an absolute value inequality and represents two linear inequalities separated by "and" (absolute value with <) or "or" (absolute value with >). ["greatOR"]

This becomes:

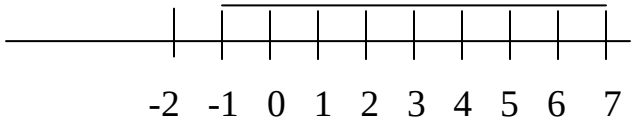
$x - 3 \leq 4$ and $x - 3 \geq -4$ Note that the inequality and negative signs are switched.

$x \leq 7$ and $x \geq -1$

With any "and" graph, you should look for the overlap and then write your answer:



The overlap is from x = -1 to x = 7 inclusive and so there is no open circles:

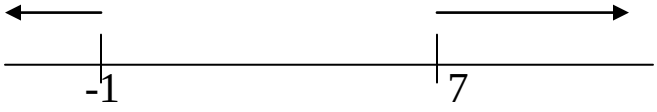
Inequality	Interval	Graph
$-1 \leq x \leq 7$	[-1,7]	

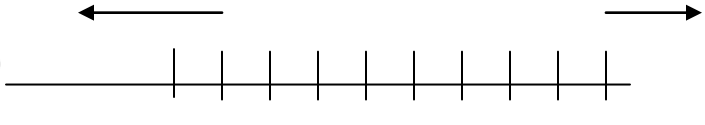
4. $|x-3| \geq 4$

This becomes:

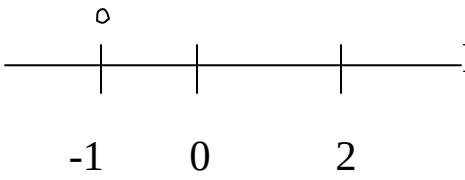
$x - 3 \geq 4$ or $x - 3 \leq -4$

$x \geq 7$ or $x \leq -1$



Inequality	Interval	Graph
$x \leq -1$ or $x \geq 7$	$(-\infty,-1] \cup [7,\infty)$	

5. $\frac{x(x-2)}{x+1} \geq 0$ This is a non-linear inequality just like example 2 and requires a number line to solve.



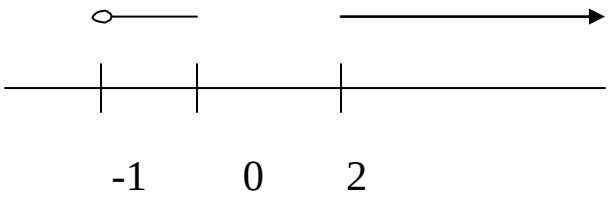
Plug into the entire fraction to see if it is positive or negative

Why is there an open circle at x = -1?

Note that an open circle was placed over the –1 to remind me even though the sign is $\geq$, x cannot be –1, since it would make the bottom zero. There are closed dots at x = 0 and x = 2.

Which regions work?

The regions that work are the second and fourth and the graph looks like:



Inequality Notation: $-1 < x \leq 0$ or $x \geq 2$

Interval Notation: $(-1,0] \cup [2,\infty)$

Homework: Ditto-Interval Notation