Pre-Calc Unit Lesson 3: Domain and Range

Domain: Set of x values

Range: Set of y values

For domain, sometimes it is easier to find what does not work:

*A square root cannot find values that are negative: For $\sqrt{x-2}$, you would have x – 2 $\geq$ 0 and thus the domain is x $\geq$ 2. Remember to use the rules in the section before this for inequalities.

*You can't divide by zero, so for a rational function, find what would make the bottom zero and state that x cannot be that number. For $\frac{x-2}{(x+1)(x-3)}$, you would have the domain: all real numbers, x ≠ -1,3.

*Logs can only use positive real numbers (not including zero). So, for log(x – 1), the domain is x – 1 > 0 and thus x > 1.

*There are other special cases, such as trig functions and you can use the graphs to help you determine the answer. You should notice that if there is not a value of x that would make the graph undefined, then the answer is all real numbers for domain.

For all ranges, you need to look at the graph and analyze

Find the domain and range for the following:

1. $y = x^2 + 1$

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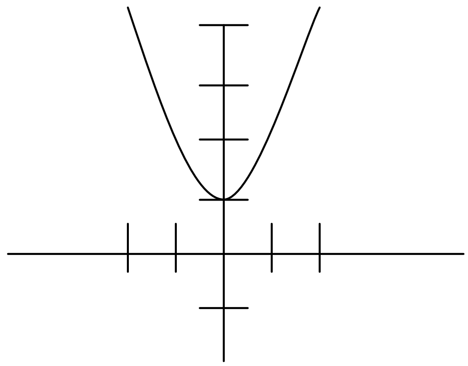
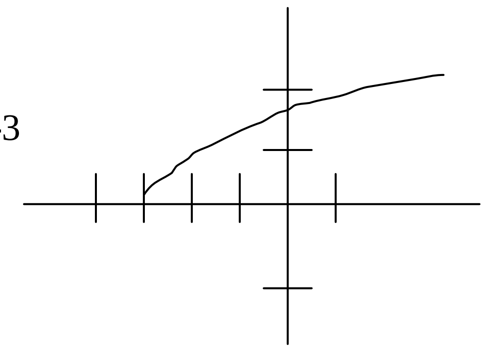


Notice for the domain, there is not a square root, rational function, or logarithm. Using the graph, you can clearly see that the range is $y \geq 1$, since there is a minimum point at (0,1)

d: all real numbers r: $y \geq 1$

2. $y = \sqrt{x+3}$



For the domain, $x + 3 \geq 0$, and so $x \geq -3$

For the range, look at the graph and determine the answer.

D: $x \geq -3$ R: $y \geq 0$

3. $y = \sqrt{x+3} + 2$

The only difference is that this graph is moved up 2 units and so algebraically, the domain stays the same and the range is up 2.

Answer:

D: $x \geq -3$ R: $y \geq 2$

4. $y = \dfrac{x-3}{x-5}$

The domain is what x cannot be and thus since 0 cannot be in the bottom of a fraction, the answer is x ≠ 5. As for the range, you need to graph this function and notice that it has a horizontal asymptote at y = 1, and thus never gets there. How can you find the asymptote without a calculator?

Look at the highest power on the top and bottom of the fraction. If it is larger on the bottom, then the asymptote is y = 0 (and thus the range is y ≠ 0). If the powers are the same, use the coefficients of those terms to find the answer. In this case, that would be 1/1 and so the horizontal asymptote is y = 1.

Answer: D: All real numbers, x ≠5 R: All real numbers, y ≠ 1

5. Try to find this one yourself: $y = \dfrac{x+2}{x^2-5x}$

d: All real numbers, x ≠ 0,5 r: All real numbers, y ≠ 0

6. $y = \sqrt{x^2-4}$

Algebraically, $x^2 - 4 \geq 0$ requires a number line to solve.

+ | - | +

-2 2

$(x - 2)(x + 2) \geq 0$

So, the domain is $x \leq -2$ or $x \geq 2$. {You can also say $(-\infty,-2] \cup [2,\infty)$}

To find the range, you need the graph. Use your graphing calculator and determine an answer:

D: $x \leq -2$ or $x \geq 2$ R: $y \geq 0$

7. $y = \sqrt{4 - x^2}$

D: [-2,2] R: [0,2] You should recognize this graph as a semicircle

8. $y = e^x + 2$

D: All real numbers R: $y > 2$ (Note e^x has a horizontal asymptote at $y = 0$ and this graph is shifted up 2 units.)

9. $y = 3\sin x$

D: All real numbers R: [-3,3]

10. $y = \tan x$

D: All real numbers, x ≠ 90 + 180k, k϶Z R: All real numbers

The tangent graph would have vertical asymptotes at the points where cosine is equal to zero, since tangent equals sine divided by cosine and you can't divide by zero. Cosine equals zero at 90,270,450, or 90 + 180k, where k is some integer.

11. Try to find the domain only for:

A. $y = \sqrt{\dfrac{x^2 - 9}{x - 9}}$

B. $y = \sqrt{x - 2} + \dfrac{1}{\sqrt{x - 3}}$

Domain: [-3,3] ∪ (9,∞)

Domain: x > 3

For A, a number line is needed with closed circles at x = 3, -3 and an open circle at x = 9, since you can't divide by zero. Once you find the region(s) that are positive, since a square root must have a positive value inside of it, you get the answer above.

For B, you get x ≥ 2 and x > 3 from the two separate pieces of the function. Note that x > 3 does not include equal to because it is on the bottom of a fraction. Since both parts would have to be true or one of the square roots would contain an imaginary number, the answer is $x > 3$.

Homework:

Ditto: Domain and Range

Quiz #1 Pre-Calc Unit