Pre-Calc Unit Lesson 4: Functions

The graphs of the following parent functions should be memorized:

$y = x^2$ $y = x^3$ $y = \sqrt{x}$ $y = |x|$

Knowing these graphs along with the following rules, you can graph several different functions:

$y = c\, f(x + a) + b$

a- moves the graph left and right and goes the opposite way you expect. If it is (x – 3), then the graph moves right 3. If (3 – x), then the graph still moves right 3 but is flipped left to right.

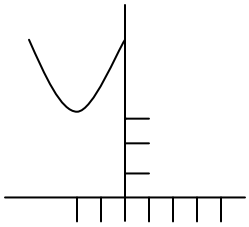
b- moves the graph up and down and follows the sign of the number. A +2 would move the graph up 2.

c- controls the steepness of the slopes of the graph. If it is negative, the graph "flips upside down".

1. $y = (x + 2)^2 + 3$

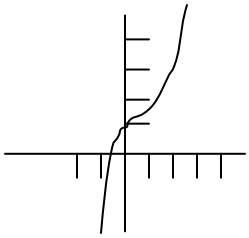
moves left 2 and up 3. It is a parabola with vertex (-2,3)

The graph looks like this:

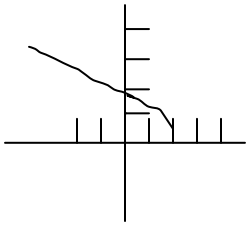


2. $y = 4x^3 + 1$

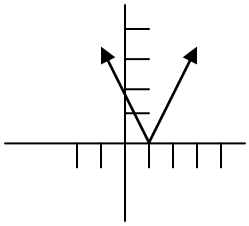
moves the cubic function up 1. The 4 does not make a noticeable difference. It only makes the slopes steeper. A number less than 1 ignoring its sign would make the slopes less steep.



3. $y = \sqrt{2 - x}$ If it was $\sqrt{x - 2}$, then the graph would be a right 2 square root function. Since 2 – x = -(x – 2), we have a reflection left to right. The graph looks like this:

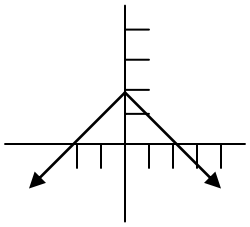


4. $y = 2|x - 1|$



The graph has a vertex at (1,0) and the slope of each line of the absolute value is now 2.

5. $y = -|x| + 2$

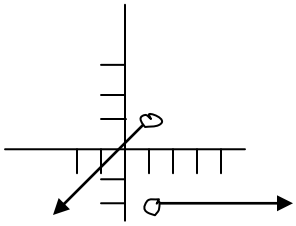


The graph has a vertex at (0,2) and the slope of each line is –1.

Find the domain, range, and zeroes of the following functions:

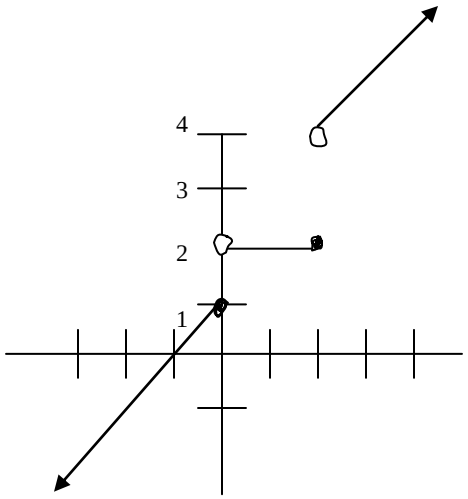
1. $f(x) = \begin{cases} x & \text{if } x < 1 \\ -2 & \text{if } x > 1 \end{cases}$

You will need to graph the function:



The domain is all real numbers, $x \neq 1$. The range is $y < 1$. The zeroes are $x = 0$ only. (Remember that zeroes are the same as x-intercepts, roots, or solutions and so you are only finding where the graph crosses $y = 0$.)

2. $g(x) = \begin{cases} x + 1 & \text{if } x \leq 0 \\ 2 & \text{if } 0 < x \leq 2 \\ x + 2 & \text{if } x > 2 \end{cases}$



The domain is all real numbers

The range is $(-\infty,1] \cup \{2\} \cup (4,\infty)$ The $\{2\}$ can be $[2,2]$

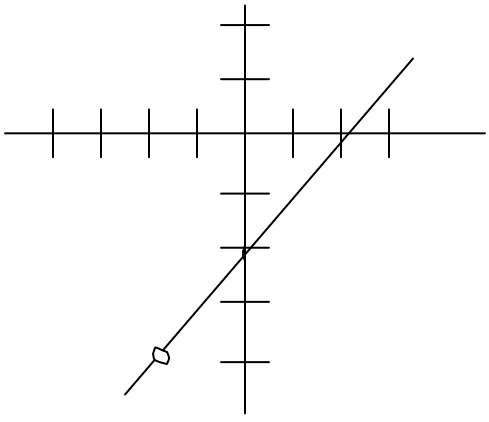
The zero is at x = -1

Part 3:

Graph $f(x) = \dfrac{x^2 - 4}{x + 2}$

Note that $\dfrac{x^2 - 4}{x + 2} = \dfrac{(x+2)(x-2)}{x+2} =$ x – 2 , x ≠ -2 and so the graph of f(x) is the same as the graph of the line x – 2 with a "hole" (removable discontinuity) in it at x = -2. We need to find what the coordinate point would be at the open circle. In other words, where would the dot have been on the line y = x – 2?

Ans: (-2,-4)



Part 4: The Greatest Integer Function:

y = [x] or y = INT(x)

The greatest integer function represents the greatest integer less than or equal to x. It is not a rounding function!

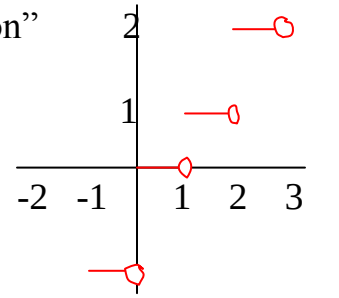
Examples:

[2] [1.7] [-1.8]

2 1 -2

What does the graph look like?

"The Step Function"



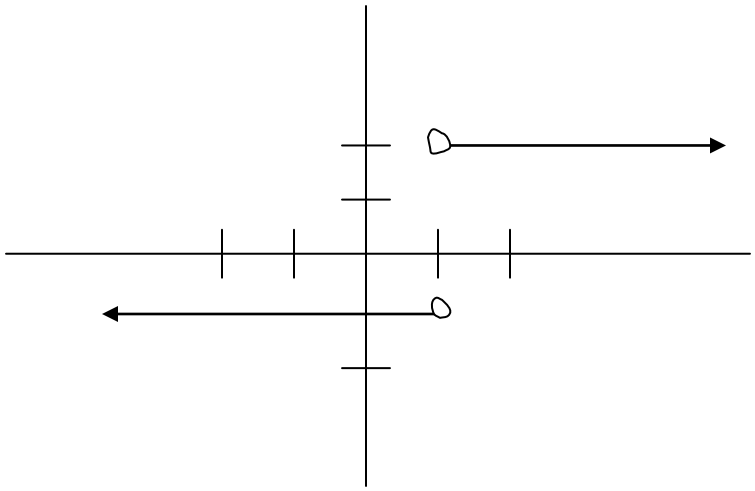
You should see why this is called the step function. For example, make an x/y chart using all the numbers between x = 1 and x = 2. If x =1, then y = 1. If x = 1.1, then y = 1. If x = 1.2, then y = 1 …… If x = 1.9, then y = 1. If x = 2, then y = 2 and thus there is an open circle at (1,2) and a closed circle at (2,2)

Part 5:

Graph $y = \frac{|x-1|}{x-1}$

Make an x/y chart;

x	y
-2	-1
-1	-1
0	-1
1	Und
2	1
3	1



The value is always either 1 or –1 except at x = 1 where the graph is undefined.

Re-write this function as a piecewise definition and try to explain where it comes from.

$$F(x) = \begin{cases} 1 & \text{if } x > 1 \\ -1 & \text{if } x < 1 \end{cases}$$

Homework: Ditto Functions (Calc)