

For Parts A-E and G, refer to the notes and answers on the school website if you get stuck.

**A. Piecewise Functions: (Do not use a calculator.)**

Name \_\_\_\_\_

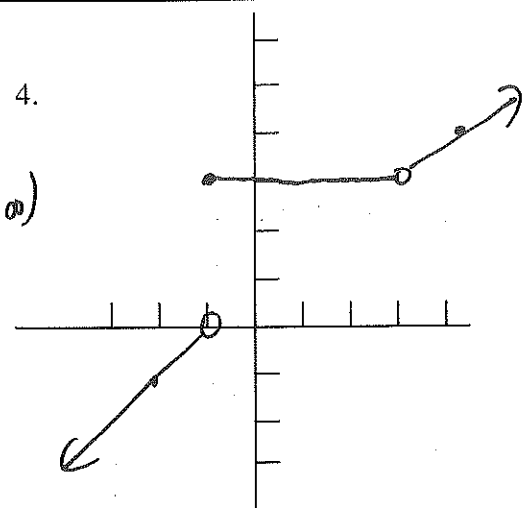
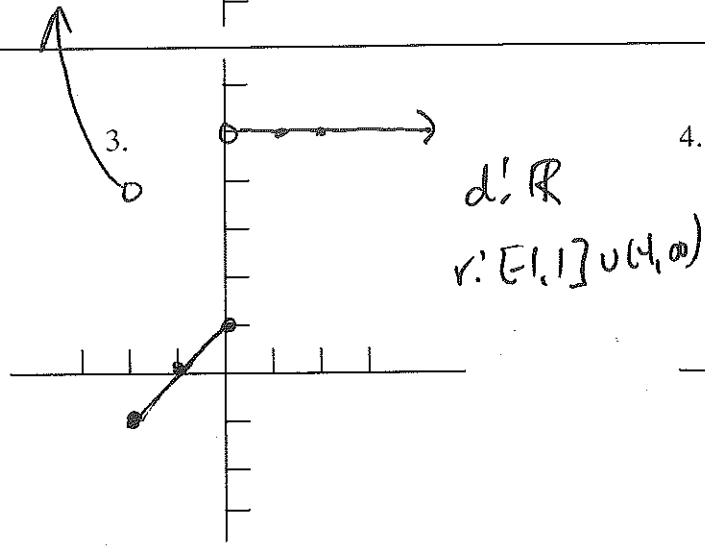
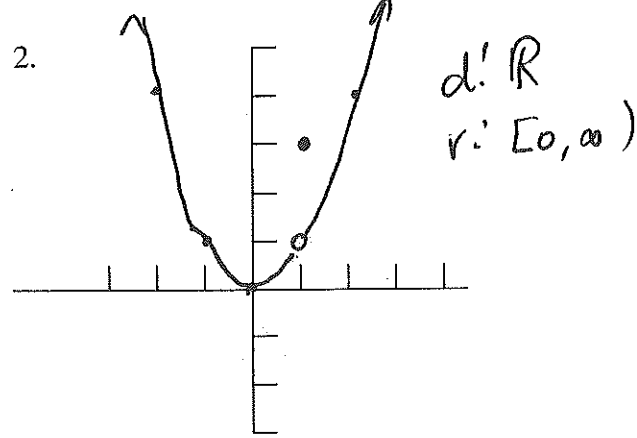
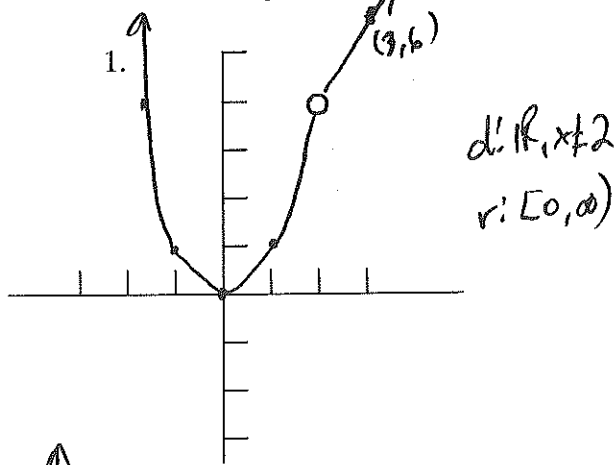
Graph the following piecewise functions:

1.  $f(x) = \begin{cases} x^2 & \text{if } x < 2 \\ 2x & \text{if } x > 2 \end{cases}$

2.  $g(x) = \begin{cases} x^2 & \text{if } x \neq 1 \\ 3 & \text{if } x = 1 \end{cases}$

3.  $f(x) = \begin{cases} 5 & \text{if } x > 0 \\ x+1 & \text{if } -2 \leq x \leq 0 \\ x^2 & \text{if } x < -2 \end{cases}$

4.  $g(x) = \begin{cases} x+1 & \text{if } x < -1 \\ 3 & \text{if } -1 \leq x < 3 \\ x & \text{if } x > 3 \end{cases}$



**B. Interval Notation/Inequalities: (Do not use a calculator.)**

Change from inequality notation to interval notation:

1.  $-2 < x \leq 1$

2.  $x > 3$  and  $x < 12.4$

3.  $x < 3$  or  $x \geq 10$

4. All real numbers except 2 and 4

5.  $x > 4$  and  $x < 2$

6.  $x \leq 6$  and  $x > -3$

2  $(3, 12.4)$

4  $(-\infty, 2) \cup (2, 4) \cup (4, \infty)$

6  $(-3, 6]$

1  $(-2, 1]$

3  $(-\infty, 3] \cup [10, \infty)$

5  $\emptyset$

Change from interval notation to inequality notation: ( $\cup$  -- or,  $\cap$  -- and)

7.  $(-3, 4]$

$-3 < x \leq 4$

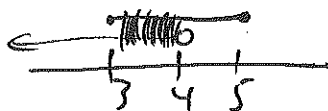
8.  $(-\infty, 4) \cup [10, \infty)$

$x < 4$  OR  $x \geq 10$

9.  $(-\infty, -2) \cup (-2, \infty)$

$\mathbb{R}, x \neq -2$  (Also:  $(x < -2$  OR  $x > -2)$ )

10.  $(-\infty, 4) \cap [3, 5]$



$3 \leq x < 4$

Solve the following inequalities. Give answer in both forms:

	Inequality form	Interval Notation	
11. $x + 7x - 8 \leq 0$	<u><math>x \leq 1</math></u>	<u><math>(-\infty, 1]</math></u>	
12. $x^2 + 7x - 8 \leq 0$	<u><math>-8 \leq x \leq 1</math></u>	<u><math>[-8, 1]</math></u>	$\begin{matrix} + & - & + \\ - & 8 & 1 \end{matrix}$
13. $x^2 - 3x - 4 > 0$	<u><math>x &lt; -1</math> OR <math>x &gt; 4</math></u>	<u><math>(-\infty, -1) \cup (4, \infty)</math></u>	$\begin{matrix} + & - & + \\ - & 1 & 4 \end{matrix}$
14. $x^2 - 3x + 4 > 0$	<u><math>\mathbb{R}</math></u>	<u><math>(-\infty, \infty)</math></u>	$\begin{matrix} + \\ - \end{matrix}$
15. $x - 3x - 4 \geq 0$	<u><math>x \leq -2</math></u>	<u><math>(-\infty, -2]</math></u>	
16. $ x - 2  < 5$	<u><math>-3 &lt; x &lt; 7</math></u>	<u><math>(-3, 7)</math></u>	
17. $ x - 2  > 5$	<u><math>x &lt; -3</math> OR <math>x &gt; 7</math></u>	<u><math>(-\infty, -3) \cup (7, \infty)</math></u>	
18. $2(x - 3) + 4(x - 5) \geq 7x$	<u><math>x \leq -26</math></u>	<u><math>(-\infty, -26]</math></u>	
19. $\frac{5(x+2)}{(x-1)(x-3)} \geq 0$	<u><math>-2 \leq x &lt; 1</math> OR <math>x &gt; 3</math></u>	<u><math>[-2, 1) \cup (3, \infty)</math></u>	$\begin{matrix} - & 0 & + \\ - & 1 & 3 \\ - & 2 & 1 \end{matrix}$

**C. Domain/Range : (Do not use a calculator. Skip the range in #5, 6, 8, 15.)**

Find the domain and range for the following functions:

	Domain	Range
1. $y = x^2 + 5$	<u><math>\mathbb{R}</math></u>	<u><math>[5, \infty)</math></u>
2. $y = \sqrt{x+5}$	<u><math>x \geq -5</math></u>	<u><math>[0, \infty)</math></u>
3. $y = \sqrt{x^2 - 9}$	<u><math>x \leq -3</math> OR <math>x \geq 3</math></u>	<u><math>[0, \infty)</math></u>
4. $y = \frac{1}{x-3}$	<u><math>\mathbb{R}, x \neq 3</math></u>	<u><math>\mathbb{R}, y \neq 0</math></u>
5. $y = \frac{5}{x^2 + 3x - 4}$	<u><math>\mathbb{R}, x \neq -4, 1</math></u>	<u>XXXXXXXXXXXX</u>
6. $y = \frac{5}{x^2 + 3x}$	<u><math>\mathbb{R}, x \neq 0, -3</math></u>	<u>XXXXXXXXXXXX</u>

$$7. y = \frac{2x-5}{x-3}$$

$$\mathbb{R}, x \neq 3$$

$$\mathbb{R}, y \neq 2$$

$$8. y = \frac{5}{x^2+3}$$

$$\mathbb{R}$$

XXXXXXXXXXXXX

$$9. y = \sqrt{x^2-3x}$$

$$x \leq 0 \text{ OR } x \geq 3$$

$$[0, \infty)$$

$$10. y = \sqrt{16-x^2}$$

$$[-4, 4]$$

$$[0, 4]$$

$$11. y = e^{x+1}$$

$$\mathbb{R}$$

$$(0, \infty)$$

$$12. y = e^x + 1$$

$$\mathbb{R}$$

$$(1, \infty)$$

$$13. y = 5\sin x$$

$$\mathbb{R}$$

$$[-5, 5]$$

$$14. y = 5\sin x + 4$$

$$\mathbb{R}$$

$$[-1, 9]$$

$$15. y = \sqrt{\frac{x}{x^2-9}}$$

$$(-3, 0] \cup (3, \infty)$$

XXXXXXXXXXXXX

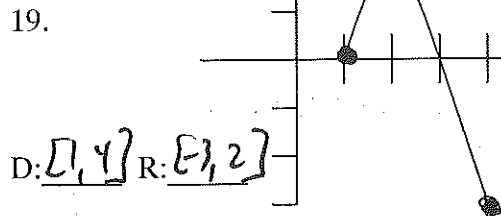
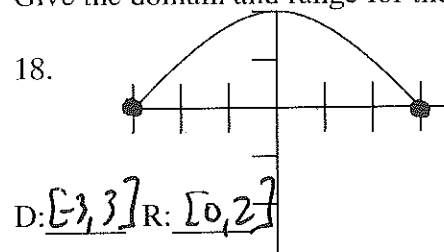


Give me a function whose domain is the following:

16. All real numbers except 2  $y = \frac{1}{x-2}$

17.  $x \geq 5$   $y = \sqrt{x-5}$

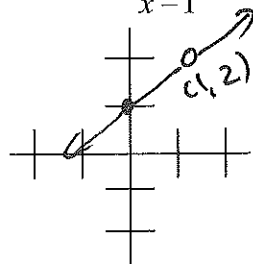
Give the domain and range for the functions graphed below:



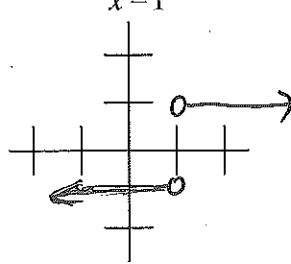
**D. Graphs of functions: (Do not use a calculator.)**

1. Graph the following:

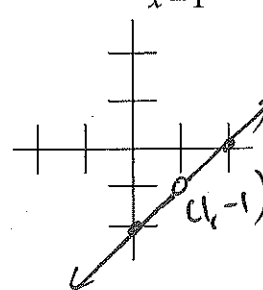
A.  $y = \frac{x^2-1}{x-1} = x+1, x \neq 1$



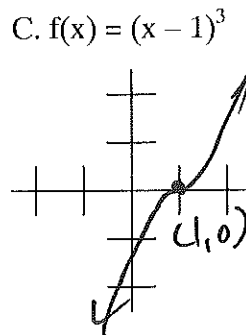
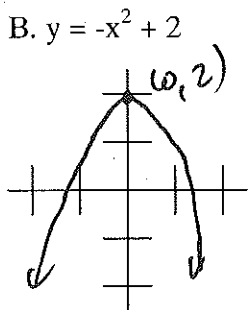
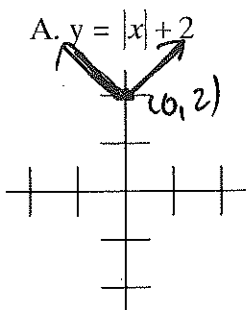
B.  $y = \frac{|x-1|}{x-1}$



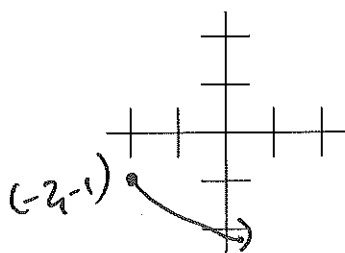
C.  $y = \frac{x^2-3x+2}{x-1} = \frac{(x-2)(x-1)}{x-1} = x-2, x \neq 1$



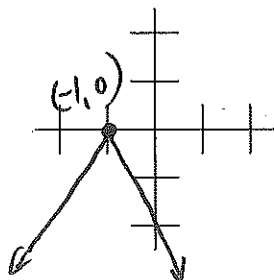
2. Sketch the following functions and label the vertex.



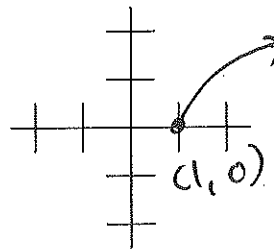
D.  $y = -\sqrt{x+2} - 1$



E.  $y = -2|x+1|$



F.  $f(x) = \sqrt{x-1}$



3. Find the domain/range and zeroes for:

A.  $f(x) = \begin{cases} x+1 & \text{if } x \leq 0 \\ 3 & \text{if } 0 < x \leq 1 \\ x^2 & \text{if } x > 1 \end{cases}$

D:  $\mathbb{R}$

R:  $\mathbb{R}$

Z:  $x = -1$

B.  $f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ -2 & \text{if } x > 1 \end{cases}$

D:  $\mathbb{R}, x \neq 1$

R:  $[0, \infty) \cup y = -2$

Z:  $x = 0$

**E. Operations on Functions: (Do not use a calculator.)**

1. If  $f(x) = \frac{1}{x+2}$ ,  $g(x) = \frac{4}{x-3}$ , and  $h(x) = 7x + 4$ , then find:

A.  $(f+g)(x) = \frac{5x+5}{(x+2)(x-3)}$

B.  $f(g(x)) = \frac{x-3}{2x-2}$

C.  $f(h(x)) = \frac{1}{7x+6}$

D.  $(f-g)(x) = \frac{-3x-1}{(x+2)(x-3)}$

E.  $f(g(h(-1))) = \frac{3}{4}$

F.  $f\left(\frac{1}{g(2)}\right) = \frac{4}{7}$

2. If  $f(x) = 2x - 3$ ,  $g(x) = x^2 - 2$ , and  $i(x) = \sqrt{x} + 1$ , then find:

A.  $f(a+1) = \underline{2a-1}$

B.  $g(a+1) = \underline{a^2+2a-1}$

C.  $f(g(2)) = \underline{1}$

D.  $g(i(9)) = \underline{14}$

E.  $(f+g)(2) = \underline{3}$

F.  $(f \cdot g)(2) = \underline{2}$

G.  $(f/g)(2) = \underline{\frac{1}{2}}$

H.  $(f-i)(1) = \underline{-3}$

I.  $g(g(x)) = \underline{x^4 - 4x^2 + 2}$

J.  $f(f(x)) = \underline{4x-9}$

K.  $f(g(x)) = \underline{2x^2-7}$

L.  $g(f(x)) = \underline{4x^2-12x+7}$

M.  $g(i(x)) = \underline{x+2\sqrt{x}-1}$

N.  $f(g(i(4))) = \underline{11}$

O.  $(f-g)(x) = \underline{-x^2+2x-1}$

P.  $f(g(i(x))) = \underline{2x+4\sqrt{x}-5}$

Q.  $\frac{f(x+h)-f(x)}{h} = \underline{2}$

R.  $\frac{g(x+h)-g(x)}{h} = \underline{2x+h}$

3. If  $f(x) = x^2 + 3x + 5$ , then find:

A.  $\frac{f(x+h)-f(x)}{h} = \underline{2x+3+h}$

B.  $\frac{f(3+h)-f(3)}{h} = \underline{9+h}$

4. If  $f(x) = 2x - 3$  and  $g(x) = (x + 3)/2$ , then find:

A.  $f(g(x)) = \underline{x}$

B.  $g(f(x)) = \underline{x}$

C.  $f(g(7000)) = \underline{7000}$

D.  $(f+g)(x) = \underline{\frac{5x-3}{2}}$

E.  $f(x)$  and  $g(x)$  share a special relationship. These functions are inverses of each other.

**F. Trig Review: (Only use your calculator on #6 and #7. All identities given below are not required to be memorized.)**

Given Identities:  $\sin(2x) = 2\sin x \cos x$

$\cos(2x) = \cos^2 x - \sin^2 x$

$\sin(A+B) = \sin A \cos B + \sin B \cos A$

$\sin(A-B) = \sin A \cos B - \sin B \cos A$

$\cos(A+B) = \cos A \cos B - \sin B \sin A$

$\cos(A-B) = \cos A \cos B + \sin B \sin A$

1. Change  $-\pi/4$  to degrees and find a positive coterminal angle.

$-\frac{\pi}{4} = \underline{-45^\circ}$

$\underline{315^\circ}$

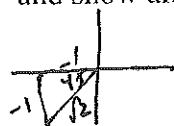
2. If  $\tan x = 3$  in quadrant I, then find the other trig functions: (Hint: Draw a triangle)

$$\sin x = \frac{3\sqrt{10}}{10} \quad \cos x = \frac{\sqrt{10}}{10} \quad \csc x = \frac{\sqrt{10}}{3} \quad \sec x = \sqrt{10} \quad \cot x = \frac{1}{3}$$

3. Find the following values without a calculator: Draw the 30/60/90 and 45/45/90 triangles in the margin.

$$\begin{aligned} \cos(\pi/3) &= \frac{1}{2} & \tan 45^\circ &= \frac{1}{1} & \sin 60^\circ &= \frac{\sqrt{3}}{2} \\ \sin(\pi) &= 0 & \sin(2\pi/3) &= \frac{\sqrt{3}}{2} & \cos(5\pi/6) &= \frac{-\sqrt{3}}{2} \end{aligned}$$

4. Find  $\cos 225^\circ$  and show all work. (Show the angle drawn in the correct quadrant.)



$$\cos 225^\circ = -\frac{1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$$

5. Find 2 values of  $x$  on  $[0^\circ, 360^\circ]$ , such that  $\sin x = 1/2$ .

$$x = 30^\circ$$

I —  
II —

$$x = 30^\circ, 150^\circ$$

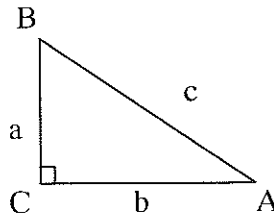
6. Find 2 values of  $x$  on  $[0^\circ, 360^\circ]$ , such that  $\cos x = 0.8123$  (Use your calculator)

$$x = \cos^{-1}(0.8123) \quad x = 35.679^\circ$$

$$\begin{aligned} &\text{II } 144.321^\circ \\ &\text{III } 215.679^\circ \end{aligned}$$

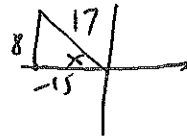
7. Find the rest of the triangle if  $A = 40^\circ$ ,  $C = 90^\circ$  and  $b = 6$  (Use calculator)

$$\begin{aligned} a &= 5.035 & A &= 40^\circ \\ b &= 6 & B &= 50^\circ \\ c &= 7.832 & C &= 90^\circ \end{aligned}$$



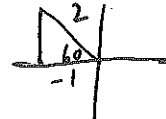
8. If  $\cos x = -15/17$  and  $\tan x < 0$ , find  $\csc x$ .

$$\frac{+17}{8}$$



9. Evaluate  $\cos 120^\circ$  using its reference angle. Show your drawing:

$$\cos 120^\circ = -\frac{1}{2}$$



10. In what 2 quadrants is  $\cot$  negative? II IV

11. Use trig identities to simplify the following trig expressions:

A.  $\tan^2 x - \tan^2 x \sin^2 x = \frac{\sin^2 x}{\cos^2 x}$

B.  $(\cos x - \sin x)^2 = 1 - 2\sin x \cos x$  or  $1 - \sin(2x)$

$$\tan^2 x [1 - \sin^2 x] = \tan^2 x [\cos^2 x] = \frac{\sin^2 x}{\cos^2 x} [\cos^2 x] = \frac{\sin^2 x}{1} \quad (\cos x - \sin x)^2 = \cos^2 x - 2\sin x \cos x + \sin^2 x = 1 - 2\sin x \cos x$$

12. True or false: Explain if false.

A.  $\sin x \cdot \csc y = 1$  FALSE

B.  $3\sec x = \frac{1}{3\cos x}$  FALSE

(unless  $x=y$ )  
 $\sin x \cdot \frac{1}{\sin y} \neq 1$

$$3\sec x = 3 \cdot \frac{1}{\cos x} = \frac{3}{1} \cdot \frac{1}{\cos x} = \frac{3}{\cos x}$$

13. Solve for values on  $[0^\circ, 360^\circ]$  for  $2\sin x + 1 = 0$

$$\sin x = -1/2$$

$$x = 210^\circ, 330^\circ$$

14. Solve  $2\sin x + \sin(2x) = 0$  on  $[0, 2\pi]$

$$2\sin x + 2\sin x \cos x = 0$$

$$2\sin x [1 + \cos x] = 0$$

$$x = 0, \pi$$

15. If  $\cos x = 5/13$  and  $270^\circ < x < 360^\circ$ , then find:

A.  $\sin(2x) = \frac{-120}{169}$

B.  $\cos(2x) = \frac{-119}{169}$

16. Find  $\cos 15^\circ$  exactly. (Hint  $60 - 45 = 15$ )

$$\frac{\sqrt{6} + \sqrt{2}}{4}$$

**G. Simplifying Complex Fractions and Rational Expressions: (Do not use a calculator.)**

Simplify the following (Find a common denominator in #10, 18-20)

1.  $\frac{3}{\frac{x}{y}} = \frac{3}{xy}$

2.  $\frac{\frac{3}{x}}{\frac{y}{x}} = \frac{3}{y}$

3.  $5x^{-2} = \frac{5}{x^2}$

4.  $(5x)^{-2} = \frac{1}{25x^2}$

5.  $\frac{\frac{5}{x} + 3}{y} = \frac{5+3x}{xy}$

6.  $(9x^2)^{\frac{1}{2}} = \frac{1}{3x}$

7.  $(\sin x) \cdot x + x^2 = x \sin x + x^2$

8.  $(\sin x) \cdot x \cdot \cos x \cdot \sin x = x \sin^2 x \cos x$

9.  $\sqrt{x} \left( x + x^{-\frac{1}{2}} \right) = \frac{x^{3/2} + 1}{x\sqrt{x} + 1}$  or

10.  $7x^{\frac{1}{2}} + 3x^{\frac{1}{2}} = \frac{7+3x}{\sqrt{x}}$

11.  $\frac{4x(2x-5) - (x^2-5x) \cdot (4)}{(2x-5)^2} = \frac{4x^2}{(2x-5)^2}$

12.  $\frac{4(x-3) \cdot 2x - 5x \cdot \left( \frac{1}{x+3} \right) + 24x}{y} =$

$$\frac{8x^3 + 24x^2 - 5x}{xy + 3y}$$

13.  $(\sin x)(\cos x)(\sec x) \cdot 7 + x^2(\tan x)(\sec^2 x) \cdot 5x =$

$$7\sin x + 5x^3 \tan x \sec^2 x$$

$$14. x \cdot e^x \cdot \frac{1}{2} x^{\frac{1}{2}} - \sqrt{x} \sin x \cdot (-3x^{\frac{1}{2}}) =$$

$$\frac{\sqrt{x} e^x + 3x \sin x}{2} = \frac{\sqrt{x} e^x + 6x \sin x}{2}$$

$$15. \frac{\frac{7x}{y} + \frac{2}{a}}{x+3} =$$

$$\frac{7ax + 2y}{axy + 3ay}$$

$$16. \frac{\frac{7x}{y} - \frac{5}{3x}}{10} = \frac{21x^2 - 5y}{30xy}$$

$$17. \frac{\frac{2x}{3} - y^{-1}}{8 - \frac{2}{y}} =$$

$$\frac{2xy - 3}{24y - 6}$$

$$18. \frac{4x}{2x-5} - \frac{3}{x-2} =$$

$$\frac{4x^2 - 14x + 15}{(2x-5)(x-2)}$$

$$19. \frac{7}{a-3} - 4 =$$

$$\frac{-4a+19}{a-3}$$

$$20. \frac{5}{\sqrt{z+1}} + \sqrt{z+1} =$$

$$\frac{z+6}{\sqrt{z+1}}$$

### H. Graphing Calculator Exercises: (A graphing calculator is required.)

1. Enter the following in the calculator exactly as written. Write each in standard notation and give the answer to each. (Note the use of parenthesis in the graphing calculator.)

A.  $-3^2 = -9$      $-3^2$     B.  $(-3)^2 = 9$      $(-3)^2$     C.  $\sqrt{1+3} = 2$      $\sqrt{1+3}$

D.  $\sqrt{1+3} = 4$   
 $\sqrt{1+3}$

E.  $1/3 + 5 = 16/3$   
 $1/3 + 5$

F.  $1/(3+5) = 1/8$   
 $1/3+5$

2. Add  $1/25 + 3/64$  in the calculator. (Enter them in as you see them with the divide key. Once you get the decimal, use the key under the MATH menu FRAC, which will change it to a fraction. On some graphing calculators, if you use 2nd and divide instead of divide, it will make it a fraction.)

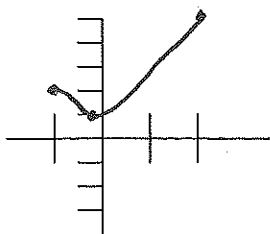
decimal: 0.086875    fraction:  $\frac{139}{1600}$

3. Find  $\sin 90$  in both degrees and radians. (In the MODE menu, there is a place to change from degrees to radians. Just scroll down to the appropriate one and hit enter.)

$\sin 90$  (in degrees) = 1     $\sin 90$  (in radians) = 0.894

4. Graph  $y = x^2 + 1$  with  $x$  on  $[-1, 2]$  and  $y$  on  $[-5, 5]$ . (Use the WINDOW key and set  $x_{\min} = -1$ ,  $x_{\max} = 2$ ,  $x_{\text{scl}} = 1$ ,  $y_{\min} = -5$ ,  $y_{\max} = 5$  and  $y_{\text{scl}} = 1$ . Then, hit the Y= key and enter in the function. Press graph.)

Draw the picture here:





5. Get a table of values for the function  $y = x^2 - 3$  starting at 1 and going to 2, skipping by 0.1 each time. (Hit the TABLESET button and enter in starting table= 1 and  $\Delta$ table = 0.1. Then enter the function above with the Y= key and push 2nd and GRAPH, which should be the TABLE button.)

Make your chart here:

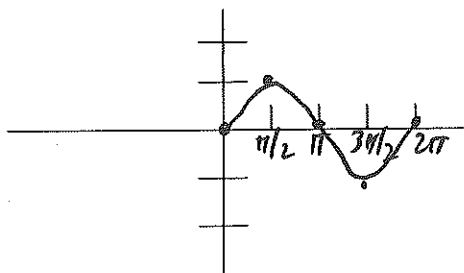
x	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
y	-2	-1.79	-1.56	-1.31	-1.04	-0.75	-0.44	-0.11	0.24	0.61	1

6. I want to know the 2 roots of  $y = x^2 - 3$  to the nearest hundredths place. (Graph the function on the interval  $[-5,5]$  by  $[-5,5]$ , by changing the window.) Use the zero feature: Once the graph is on the screen, select ZERO under the CALC (2ND and TRACE) menu. It is #2 on TI-83's. It will ask for a left bound and then a right bound and finally guess. Pick a value to the left and then to the right of the root. Lastly, put the cursor as close to the root as possible when prompted guess. (Remember the root is where the graph touches the x-axis and is the solution to the equation  $x^2 - 3 = 0$ .) Give 3 places after the decimal point for accuracy.

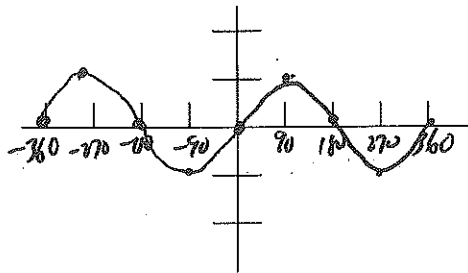
Answers: 1.732 -1.732

7. Graph  $y = \sin x$  on two different intervals by changing the WINDOW to the x by y given:

A. in radians on  $[0, 2\pi]$  by  $[-2, 2]$ ,  
with XSCL =  $\pi/2$  and YSCL = 1



B. in degrees on  $[-360, 360]$  by  $[-2, 2]$   
with XSCL = 90 and YSCL = 1



8. By graphing the following functions, give their domain and range:

\*A.  $y = \sqrt{9 - 9x^2}$  {Enter in as  $\sqrt{9 - 9x^2}$ }      B.  $y = \sqrt{x - 3} + 1$  {Enter  $\sqrt{(x-3)+1}$ }

d:  $[-1, 1]$       r:  $[0, 3]$

d:  $[3, \infty)$       r:  $[1, \infty)$

C.  $y = \sqrt{3 - x} + 1$

d:  $[-\infty, 3]$       r:  $[1, \infty)$

D.  $y = x^2 - 4x + 5$

d:  $\mathbb{R}$       r:  $[1, \infty)$

\*For 8a, zoom in on the graph by adjusting the window until you see the semi-ellipse touch the x-axis twice. Use your knowledge of domain and range to check your work. No decimal answers.)

9. Find the roots of  $y = x^2 - 16$  (Use the ZERO button)

$$\underline{x = 4, -4}$$

10. Find the roots of  $y = x^3 + x^2 - 17$  (Use the ZERO button. Give 3 places after the decimal point for accuracy.)

$$\underline{x = 2.277}$$

11. . Solve for at least one solution of each of the equations below using your calculator. All answers must be accurate to three decimal places and should include a quick sketch. Your calculators must be in radians. For the third one, get it equal to zero first. (Use the zero button. Give 3 places after the decimal point for accuracy.)

A.  $x^7 - 11 = 0$

B.  $\sin(x^2) + x - \cos x = 0$

C.  $\cos x = \sin x + x$

$$\underline{x = 1.409}$$

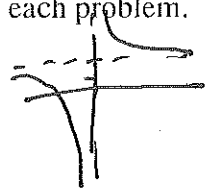
$$\underline{x = 0.552}$$

$$\underline{x = 0.457}$$

12. Graph the following functions and tell the difference the parenthesis makes in each problem. Rewrite the equations without parenthesis in standard notation:

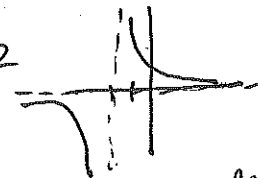
A.  $y = 1/x + 2$        $y = \frac{1}{x} + 2$

A. VA at  $x = 0$



B.  $y = 1/(x + 2)$        $y = \frac{1}{x+2}$

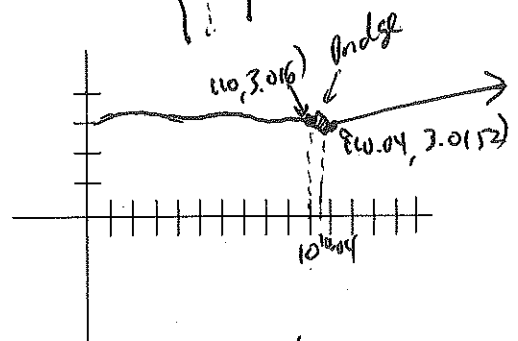
B. VA at  $x = -2$



13. The profile of a road in miles is given by  $P(x)$  below.

$$P(x) = \begin{cases} 3 + .04\cos(2x) & \text{if } 0 \leq x \leq 10 \\ \text{Bridge!!!!} & \text{if } 10 < x < 10.04 \\ 3.02 + 0.005(x - 11) & \text{if } x \geq 10.04 \end{cases}$$

A. Sketch the road.



B. What is the height of the bridge on each side?  
(Give 3 places after the decimal point for accuracy.)

$$\underline{3.016 \quad 3.0152}$$

C. Approximate the slope of the bridge.  
(Give 3 places after the decimal point for accuracy.)

$$\underline{-0.020}$$

$$m = \frac{3.016 - 3.0152}{10 - 10.04}$$